Low Energy Compton Scattering and Nucleon Structure

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February 9, 2008

Abstract

The low energy virtual Compton scattering process $eN \to e'N\gamma$ offers a new and potentially high resolution window on nucleon structure via measurement of so-called generalized polarizabilities (GPs). We present calculations of GPs within heavy baryon chiral perturbation theory and discuss present experimental efforts.

1 (En)Lightning Real Compton Review

The physics of (real) Compton scattering has received a good deal of recent attention and it is useful, before plunging into the virtual case, to have a quick review of some of the interesting issues in RCS. One of the primary goals of contemporary particle/nuclear physics is to understand the structure of the nucleon. Indeed this is being pursued at the very highest energy machines such as SLAC and HERMES, wherein one probes the quark/parton substructure, as well as at lower energy accelerators such as MAMI and BATES, wherein one studies behavior of the nucleon in terms of a collective three quark mode. In recent years one of the important low energy probes has been Compton scattering, by which one can study the deformation of the nucleon under the influence of quasi-static electric and/or magnetic fields.[1] For example, in the presence of an external electric field $\vec{E}$ the quark distribution of the nucleon becomes distorted, leading to an induced electric dipole moment

$$\vec{p} = 4\pi\alpha_E\vec{E} \tag{1}$$

in the direction of the applied field, where α_E is the electric polarizability. The interaction of this dipole moment with the field leads to a corresponding interaction energy

$$U = -\frac{1}{2}4\pi\alpha_E\vec{E}^2 \tag{2}$$

Similarly in the presence of an applied magnetizing field $\vec{H}$ there will be in induced magnetic dipole moment and interaction energy

$$\vec{\mu} = 4\pi\beta_M\vec{H}, \quad U = -\frac{1}{2}4\pi\beta_M\vec{H}^2 \tag{3}$$

For wavelengths large compared to the size of the system, the effective Hamiltonian for the interaction of a system of charge e and mass m with an electromagnetic field is, of course, given by the simple form

$$H^{(0)} = \frac{(\vec{p}-e\vec{A})^2}{2m} + e\phi \tag{4}$$

As the energy increases, however, one must also take into account polarizability effects and the effective Hamiltonian becomes

$$H_{\text{eff}} = H^{(0)} - \frac{1}{2}4\pi(\alpha_E\vec{E}^2 + \beta_M\vec{H}^2) \tag{5}$$

The Compton scattering cross section from such a system (taken, for simplicity, to be spinless) is then given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha_{em}}{m}\right)^2\left(\frac{\omega}{\omega'}\right)^2[\frac{1}{2}(1+\cos^2\theta) - \frac{m\omega\omega'}{\alpha_{em}}[\frac{1}{2}(\alpha_E+\beta_M)(1+\cos\theta)^2 + \frac{1}{2}(\alpha_E-\beta_M)(1-\cos\theta)^2+\ldots] \quad (6)$$

where α_{em} is the fine structure constant and ω, ω' are the initial, final photon energies respectively. It is clear from Eq. 6 that careful measurement of the differential scattering cross section allows extraction of these structure dependent polarizability terms *provided* that i) the energy is large enough that such terms are significant compared to the leading Thomson piece and ii) that the energy is not too large that higher order corrections become important. In this way the measurement of electric and magnetic polarizabilities for the proton has recently been accomplished using photons in the energy range 50 MeV $< \omega <$ 100 MeV, yielding[2]

$$\alpha_E^p = (12.1 \pm 0.8 \pm 0.5)\times 10^{-4}\mathrm{fm}^3, \quad \beta_M^p = (2.1 \mp 0.8 \mp 0.5)\times 10^{-4}\mathrm{fm}^3 \quad (7)$$

¿From these results, which say that the polarizabilities of the proton are nearly a factor of a thousand smaller than the corresponding nucleon volume, we learn that the nucleon is a rather rigid object when compared to the hydrogen atom, for example, wherein the electric polarizability and volume are comparable.

Additional structure probes are possible if we exploit the feature of nucleon spin.[3] Thus, for example, the presence of a time varying electric field in the plane of a rotating system of charges will lead to a charge separation and induced electric dipole moment

$$\vec{p} = -\gamma_1\vec{S}\times\frac{\partial\vec{E}}{\partial t} \quad (8)$$

with corresponding interaction energy

$$U_1 = -\vec{p}\cdot\vec{E} = \gamma_1\vec{E}\cdot\vec{S}\times(\vec{\nabla}\times\vec{B}) \quad (9)$$

where we have used the Maxwell equations in writing this form. (Note that the "extra" time or spatial derivative is required by time reversal invariance since $\vec{S}$ is T-odd.) Similarly other possible structures are

$$U_2 = \gamma_2\vec{B}\cdot\vec{\nabla}\vec{S}\cdot\vec{E}, \quad U_3 = \gamma_3\vec{E}\cdot\vec{\nabla}\vec{S}\cdot\vec{B}, \quad U_4 = \gamma_4\vec{B}\cdot\vec{S}\times(\vec{\nabla}\times\vec{E}) \quad (10)$$

and the measurement of these various "spin-polarizabilities" γ_i via polarized Compton scattering provides a rather different sort of probe for nucleon structure. Because of the requirement for polarization not much is known at present about such spin-polarizabilities, although from dispersion relations the combination[4]

$$\gamma_0^p \equiv \gamma_1^p - \gamma_2^p - 2\gamma_4^p \approx -1.34 \times 10^{-4}\text{fm}^4 \tag{11}$$

has been calculated and from a global analysis of unpolarized Compton data, to which it contributes in higher orders, one has determined the so-called backward polarizability to be[5]

$$\gamma_\pi = \gamma_1 + \gamma_2 + 2\gamma_4 = (27.7 \pm 2.3 \pm 2.5) \times 10^{-4}\text{fm}^4 \tag{12}$$

Clearly such measurements represent an important goal for the future.

2 Virtual Compton Scattering: Formalism

Recently a new frontier in Compton scattering has been opened (see, *e.g.*, [6, 7]) and is in the beginning of being explored: the study of the electron scattering process $ep \to e'p'\gamma$ in order to obtain information concerning the virtual Compton scattering (VCS) process $\gamma^* N \to \gamma N$. As will be discussed below, in addition to the two kinematical variables of real Compton scattering—the scattering angle θ and the energy ω' of the outgoing photon—the invariant structure functions for VCS [8],[9] depend on a *third* kinematical variable—the magnitude of the three–momentum transfer to the nucleon in the hadronic c.m. frame, $\bar{q} \equiv |\vec{q}|$. The VCS amplitude can then, as shown by [9], be characterized in terms of structure coefficients having $\bar{q}$ dependence and are called "generalized polarizabilities" (GPs) of the nucleon in analogy to the well-known polarizability coefficients in real Compton scattering. (However, due to the specific kinematic approximation chosen in [9] there is no one–to–one correspondence between all the real Compton polarizabilities and the GPs of Guichon et al. in VCS [9, 10, 11].)

The advantage of VCS lies in the virtual nature of the initial state photon and the associated possibility of an *independent* variation of photon energy and momentum, thus rendering access to a much greater variety of structure information than in the case of real Compton scattering. For example,

one can hope to identify the individual signatures of specific nucleon resonances in the various GPs, which cannot be obtained in other processes [6]. In this regard, it should be noted that a great deal of theoretical work has taken place and predictions for both spin-independent and spin-dependent GPs are available within a non–relativistic constituent quark model [9] and a one–loop calculation in the linear sigma model [12]. In addition, various approaches have been used to calculate the two spin–independent polarizabilities $\bar{\alpha}_E(\bar{q}^2)$ and $\bar{\beta}_M(\bar{q}^2)$, namely, an effective Lagrangian approach including nucleon resonance effects [13], our calculation of the leading $\bar{q}^2$ dependence in heavy–baryon ChPT (HBChPT) [14] and a calculation of $\bar{\alpha}_E(\bar{q}^2)$ in the Skyrme model [15]. For an overview of the status at higher energies and in the deep inelastic regime we refer to [6].

The GPs of the nucleon are defined in terms of electromagnetic multipoles as functions of the initial photon momentum $\bar{q}$ [9] ,

$$\begin{aligned} P^{(\rho' L', \rho L)S}(\bar{q}^2) &= \left[\frac{1}{\omega'^L \bar{q}^L} H^{(\rho' L', \rho L)S}(\omega', \bar{q})\right]_{\omega'=0}, \\ \hat{P}^{(\rho' L', L)S}(\bar{q}^2) &= \left[\frac{1}{\omega'^L \bar{q}^{L+1}} \hat{H}^{(\rho' L', L)S}(\omega', \bar{q})\right]_{\omega'=0}, \end{aligned} \tag{13}$$

where L (L') denotes the initial (final) photon orbital angular momentum, ρ (ρ') the type of multipole transition ($0 = C$ (scalar, Coulomb), $1 = M$ (magnetic), $2 = E$ (electric)), and S distinguishes between non–spin–flip ($S = 0$) and spin–flip ($S = 1$) transitions. Mixed–type polarizabilities, $\hat{P}^{(\rho' L', L)S}(\bar{q}^2)$, have been introduced, which are neither purely electric nor purely Coulomb type. It is important to note that the above definitions are based on the kinematical approximation that the multipoles are expanded around $\omega' = 0$ and *only terms linear in ω' are retained*, which together with current conservation yields selection rules for the possible combinations of quantum numbers of the GPs. In this approximation, ten GPs have been introduced in [9] as functions of $\bar{q}^2$: $P^{(01,01)0}$, $P^{(11,11)0}$, $P^{(01,01)1}$, $P^{(11,11)1}$, $P^{(01,12)1}$, $P^{(11,02)1}$, $P^{(11,00)1}$, $\hat{P}^{(01,1)0}$, $\hat{P}^{(01,1)1}$, $\hat{P}^{(11,2)1}$.

However, recently it has been proved [10, 11] using crossing symmetry and charge conjugation invariance that only *six* of the above ten GPs are independent. Then in the scalar (i.e. spin–independent) sector it is convenient to eliminate the mixed polarizability $\hat{P}^{(01,1)0}$ in favor of $P^{(01,01)0}$ and $P^{(11,11)1}$, because the latter are generalizations of the electric and magnetic

polarizabilities in real Compton scattering:

$$\bar{\alpha}_E(\bar{q}^2) = -\frac{e^2}{4\pi}\sqrt{\frac{3}{2}}P^{(01,01)0}(\bar{q}^2)\,, \quad \bar{\beta}_M(\bar{q}^2) = -\frac{e^2}{4\pi}\sqrt{\frac{3}{8}}P^{(11,11)0}(\bar{q}^2)\,. \tag{14}$$

However, in the spin–dependent sector it is not a priori clear which three GPs should be eliminated with the help of the C-constraints.

3 Chiral Calculation of Generalized Polarizabilities

As stated above, there have been a number of theoretical approaches to calculation of the generalized polarizabilities in addition to the heavy baryon chiral perturbative study reported below. An advantage of the latter, however, is that it is guaranteed to satisfy all field theory constraints such as crossing symmetry, charge conjugation invariance, etc. In addition, the calculation at $\mathcal{O}(p^3)$ of nucleon electric and magnetic polarizabilities for the case of real Compton scattering is know to be in agreement with experiment, so one hopes that the same may hold for the GPs. Indeed the diagrams are the same. Only the kinematics is different—instead of the usual RCS variables ω', θ, there is an additional variable $|\vec{q}|$, the center of mass momentum of the incident virtual photon in the VCS case. We have evaluated the GPs using the standard formalism of HBχPT and have obtained closed form expressions for each.

We analyze the VCS process using the standard chiral perturbation theory Lagrangian in the heavy baryon formulation to $O\left(p^3\right)$ in the nucleon sector[16, 17],

$$\mathcal{L}^{\chi}_{\pi N} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N}\,, \tag{15}$$

with

$$\begin{aligned}
\mathcal{L}^{(1)}_{\pi N} &= \bar{N}_v(iv\cdot D + g_A S\cdot u)N_v\,,\\
\mathcal{L}^{(2)}_{\pi N} &= -\frac{1}{2M}\bar{N}_v\left\{D\cdot D - (v\cdot D)^2\right.\\
&- \left.\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}v^\rho S^\sigma\left[f_+^{\mu\nu}\left(1+4c_6\right) + 2v^{(s),\mu\nu}\left(1+2c_7\right)\right]\right\}N_v - [S_\mu, S_\nu][D^\mu, D^\nu]
\end{aligned}$$

$$\mathcal{L}_{\pi N}^{(3)} = \frac{1}{2M^2}\bar{N}_v\left\{\left[f_+^{\mu\nu}\left(c_6+\frac{1}{8}\right)+v^{(s),\mu\nu}\left(c_7+\frac{1}{4}\right)\right]\right. \times \left.\varepsilon_{\mu\nu\rho\sigma}S^\sigma iD^\rho + \text{h.c.}\right\}N_v\,, \tag{16}$$

where $\varepsilon_{0123}=1$. Here we keep those terms which contribute to a $O(p^3)$ VCS calculation. In particular terms linear in the photon fields, which vanish in our gauge, have been omitted. The definitions of symbols used in Eq. 16 are standard and can be found, *e.g.* in ref. [16]

Explicit forms for each of the GPs can be found in ref. [18] Here for space reasons we quote only the generalized electric and magnetic polarizabilities

$$\begin{aligned}
\bar{\alpha}_E^{(3)}(\bar{q}) &= \frac{e^2g_A^2m_\pi}{64\pi^2F_\pi^2}\,\frac{4+2\frac{\bar{q}^2}{m_\pi^2}-\left(8-2\frac{\bar{q}^2}{m_\pi^2}-\frac{\bar{q}^4}{m_\pi^4}\right)\frac{m_\pi}{\bar{q}}\arctan\frac{\bar{q}}{2m_\pi}}{\bar{q}^2\left(4+\frac{\bar{q}^2}{m_\pi^2}\right)}\,,\\
\bar{\beta}_M^{(3)}(\bar{q}) &= \frac{e^2g_A^2m_\pi}{128\pi^2F_\pi^2}\,\frac{-\left(4+2\frac{\bar{q}^2}{m_\pi^2}\right)+\left(8+6\frac{\bar{q}^2}{m_\pi^2}+\frac{\bar{q}^4}{m_\pi^4}\right)\frac{m_\pi}{\bar{q}}\arctan\frac{\bar{q}}{2m_\pi}}{\bar{q}^2\left(4+\frac{\bar{q}^2}{m_\pi^2}\right)}
\end{aligned} \tag{17}$$

The meaning of these forms can be found by expanding

$$\begin{aligned}
\bar{\alpha}_E^{(3)}(\bar{q}) &= \frac{5e^2g_A^2}{384\pi^2F_\pi^2m_\pi}\left[1-\frac{7}{50}\frac{\bar{q}^2}{m_\pi^2}+\frac{81}{2800}\frac{\bar{q}^4}{m_\pi^4}+\mathcal{O}(\bar{q}^6)\right],\\
\bar{\beta}_M^{(3)}(\bar{q}) &= \frac{e^2g_A^2}{768\pi^2F_\pi^2m_\pi}\left[1+\frac{1}{5}\frac{\bar{q}^2}{m_\pi^2}-\frac{39}{560}\frac{\bar{q}^4}{m_\pi^4}+\mathcal{O}(\bar{q}^6)\right],
\end{aligned} \tag{18}$$

We see then that at the real photon point—$\bar{q}=0$—one reproduces the usual chiral forms

$$\alpha_E(0)=10\beta_M(0)=\frac{\alpha g_A^2}{48\pi F_\pi^2m_\pi}=12.2\times10^{-4}fm^3 \tag{19}$$

in good agreement with experiment. New are the predictions for the $\bar{q}$ dependence. In the case of the electric polarizability there is nothing unexpected—one sees a gradual fall-off with momentum transfer corresponding to a size $\sim$1 fm. However, in the magnetic case, there is a surprise—the generalized magnetic polarizability is predicted to *rise* before reaching a maximum at $\bar{q}\sim100$ MeV and then falling. This behavior is given only in chiral models, and indicates the presence of contributions to the local magnetic polarizability of opposite sign. However, it is not clear at present what the physical

origin of this effect might be. In any case it will be interesting to look for experimentally, as it distinguishes chiral models from constituent quark predictions.

4 Experimental Possibilities

Of course, calculation of the generalized polarizabilities is only really interesting to the extent that such quantities can be confronted with experimental data. The challenges here are great. The problem is firstly that such effects are relatively small. In the case of RCS, for example, the interference of of the polarizability terms in the cross section gives at most $\sim 10\%$ modifications to the cross section at a photon energy of 100 MeV. However, at this energy one must already worry about modifications also coming from terms in the effective Lagrangian of order ω^4, which are estimated using dispersive methods. The same is true of generalized polarizabilities. These are *not* large effects. However, the problem is much worse. In the case of RCS, the primary background comes from Thomson scattering. However, in the case of VCS, the basic reaction is $ep \rightarrow e'p\gamma$, which means that one is sensitive both to the sought-for e, e' spectator-$\gamma^* p \rightarrow \gamma p$ reaction as well as to p, p' spectator-$\gamma^* e \rightarrow \gamma e$, *i.e.* the Bethe-Heitler process, wherein the final photon is radiated from either the initial or final state electron. Because of the lightness of the electron this bremsstrahlung process is very important and generally dominates the cross section unless one chooses the kinematic region carefully. In addition, the entire process is quite sensitive to radiative corrections, which must be calculated quite precisely.[7]

Despite these difficulties, several groups have taken up the experimental challenge. In the case of an experiment mounted at MAMI data taking has already taken place.[19] and it looks as if the group will be able to extract values for the generalized polarizabilities. The basic problem in this regard is that in-plane kinematics were employed, meaning that the (e,e') and (p,p') planes were parallel. In this case the Bethe-Heitler reaction produces two blow-torch-like peaks in the differential scattering cross section corresponding to radiation from either the initial or final state electron and the desired GP effects are small perturbations. The careful measurements of this group has been able to verify the basic correctness of the radiative correction calculations and can reproduce the data by a sum of Bethe-Heitler and nucleon

Born diagram terms. The effect of the GPs is calculated to be about 10% in the backward direction (*i.e.* when the photon is emitted oppositely to the electron directions.)

A different approach has been taken by an approved BATES experiment by the OOPS collaboration.[20] In this case the use of the movable OOPS spectrometers allow an experiment to be performed at perpendicular orientation of the electron and proton planes, whereby the influence of the Bethe-Heitler forward peaks is minimized. Theoretically one expects the Born and Bethe-Heitler contributions to make roughly equal contributions so that any additional effect from generalized polarizabilites should be possible to see. Alternatively an approved CEBAF measurement expects to get around the problem of Bethe-Heitler backgrounds by a different route.[21] Even though employing parallel kinematics, the use of the higher CEBAF beam energy means that the experiment can be performed at a larger value of longitudinal polarization—$\epsilon \approx 0.95$. In this case, the larger virtual photon flux, which scales as $1/(1-\epsilon)$ means that the VCS contribution will be corresponding magnified and calculations reveal possible 20-30 % effects coming from GPs.

In addition to these experiments a great deal of work is focussing also on higher energies and momentum transfers where one may be able to sort out the basic and angular momentum and spin structure of the nucleon itself.[6] At the present time then the VCS glass is not only full—it is overflowing!

Acknowledgement

It is a pleasure to acknowledge the support of the Alexander von Humboldt Foundation as well as the hospitality of Forchungszentrum Jülich. This work was also supported by the National Science Foundation.

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